INTERNATIONAL		BACCALAUREATE
BACCALAURÉAT	**IB**	INTERNATIONAL
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MATHEMATICAL STUDIES

Standard Level

Friday 5 November 1999 (morning)

Paper 2   1 hour 30 minutes

This examination paper consists of two sections, Section A and Section B.
Section A consists of 4 questions.
Section B consists of 3 questions.
The maximum mark for Section A is 60.
The maximum mark for each question in Section B is 20.
The maximum mark for this paper is 80.

INSTRUCTIONS TO CANDIDATES

Do NOT open this examination paper until instructed to do so.

Answer all FOUR questions from Section A and ONE question from Section B.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required:
IB Statistical Tables
Millimetre square graph paper
Calculator
Ruler and compasses

Allowed:
A simple translating dictionary for candidates not working in their own language

FORMULAE

Sine Rule:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule:
$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Arithmetic sequences:
$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + l) = \frac{n}{2}\{2a + (n-1)d\}$$

Geometric sequences:
$$u_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

Simple interest:
$$I = \frac{Crn}{100}$$

Compound interest:
$$I = C\left(1 + \frac{r}{100}\right)^n - C$$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

*A correct answer with **no** indication of the method used will normally receive **no** marks. You are therefore advised to show your working.*

SECTION A

Answer all FOUR questions from this section.

1. *[Maximum mark: 15]*

   (i) Let

   $$\mathscr{C} = \{\text{positive integers less than } 15\} \;;$$
   $$X = \{\text{multiples of } 2\} \;;$$
   $$Y = \{\text{multiples of } 3\} \;.$$

   (a) Show, in a Venn diagram, the relationship between the **sets** $\mathscr{C}$, X and Y. *[1 mark]*

   (b) List the elements of:

   (i) $X \cap Y$ *[1 mark]*

   (ii) $X \cap \complement Y$. *[2 marks]*

   (c) Find the **number of elements** in the complement of $(X \cup Y)$. *[2 marks]*

   (This question continues on the following page)

(Question 1 continued)

(ii) Three propositions are defined as follows:

p : The oven is working.
q : The food supply is adequate.
r : The visitors are hungry.

(a) Write one sentence, in words only, for each of the following logic statements.

(i) $q \wedge r \wedge \neg p$ *[2 marks]*

(ii) $\neg r \vee (p \wedge q)$ *[2 marks]*

(b) Write the sentence below using only the symbols p, q and logic connectives.

"If the oven is working and the food supply is adequate then the oven is working or the food supply is adequate." *[2 marks]*

(c) A tautology is a compound statement which is always true. Use a truth table to determine whether or not your answer to part (b) is a tautology.

Hint: Begin by writing the first two columns of your truth table in the following format:

p	q
T	T
T	F
F	T
F	F

[3 marks]

2. *[Maximum mark: 15]*

A marine biologist records as a frequency distribution the lengths (L), measured to the nearest centimetre, of 100 mackerel. The results are given in the table below.

Length of mackerel (L cm)	Number of mackerel
$27 < L \leqslant 29$	2
$29 < L \leqslant 31$	4
$31 < L \leqslant 33$	8
$33 < L \leqslant 35$	21
$35 < L \leqslant 37$	30
$37 < L \leqslant 39$	18
$39 < L \leqslant 41$	12
$41 < L \leqslant 43$	5
	100

(a) Construct a cumulative frequency table for the data in the table. *[2 marks]*

(b) Draw a cumulative frequency curve.

   Hint: Plot your cumulative frequencies at the top of each interval. *[3 marks]*

(c) Use the cumulative frequency curve to find an estimate, to the nearest cm for

   (i) the median length of mackerel; *[2 marks]*

   (ii) the interquartile range of mackerel length. *[2 marks]*

(d) It is known that the length of mackerel is normally distributed with mean 35 cm and standard deviation 5 cm. Find the probability that a mackerel chosen at random is

   (i) longer than 40 cm ; *[2 marks]*

   (ii) less than 28 cm ; *[2 marks]*

   (iii) between 28 cm and 40 cm . *[2 marks]*

3.   *[Maximum mark: 15]*

Angela needs $ 4000 to pay for a car. She was given two options by the car seller.

Option A: *Outright Loan*

A loan of $ 4000 at a rate of 12% per annum compounded monthly.

(a) Find

     (i) the cost of this loan for one year;   *[2 marks]*

     (ii) the equivalent annual simple interest rate.   *[2 marks]*

Option B: *Friendly Credit Terms*

A 25% deposit, followed by 12 equal monthly payments of $ 287.50 .

(b)   (i) How much is to be paid as a deposit under this option?   *[1 mark]*

     (ii) Find the cost of the loan under *Friendly Credit Terms*.   ·*[2 marks]*

(c) Give a reason why Angela might choose

     (i) **Option A**

     (ii) **Option B**   *[2 marks]*

To help Angela, her employer agrees to give her an interest free loan of $ 4000 to buy the car. The employer is to recover the money by making the following deductions from Angela's salary:

$\$ x$ in the first month,
$\$ y$ every subsequent month.

The total deductions after 20 months is $ 1540 and after 30 months it is $ 2140 .

(d) Find x and y.   *[4 marks]*

(e) How many months will it take for Angela to completely pay off the $ 4000 loan?   *[2 marks]*

4. *[Maximum mark: 15]*

The vectors $\overrightarrow{BA}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ form a triangle ABC. The points $A(4, 5)$, $B(6, 2)$, $C(2, 1)$ are the vertices of triangle ABC.

(a)　(i) On coordinate axes, plot and label the points A, B and C. 　　　*[2 marks]*

　　　(ii) Draw line segments representing the vectors $\overrightarrow{BA}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$. Indicate clearly the direction of each vector. 　　　*[2 marks]*

(b)　Write as column vectors

　　　(i) $\overrightarrow{CA}$; 　　　*[2 marks]*

　　　(ii) $\overrightarrow{BC}$; 　　　*[2 marks]*

　　　(iii) $\overrightarrow{BA}$. 　　　*[2 marks]*

$A'B'C'$ is the image of triangle ABC after it is rotated through 270° about the point $(0, 0)$.

(c)　(i) On the same axes as in part (a) above, draw and label triangle $A'B'C'$. 　　　*[2 marks]*

　　　(ii) Write the image of the vector $\overrightarrow{AB}$ in the form $\begin{pmatrix} p \\ q \end{pmatrix}$. 　　　*[2 marks]*

(d)　Write down the 2×2 matrix, T, that corresponds to a rotation about $(0, 0)$ through 270° . 　　　*[1 mark]*

SECTION B

Answer ONE question from this section.

5. *[Maximum mark: 20]*

Vanessa wants to rent a place for her wedding reception. She obtains two quotations.

(a) The local council will charge her £30 for the use of the community hall plus £10 per guest.

   (i) **Copy** and complete this table for charges made by the local council.

Number of guests (N)	10	30	50	70	90
Charges (C) in £					

   [2 marks]

   (ii) On graph paper, using suitable scales, draw and label a graph showing the charges. Take the horizontal axis as the number of guests and the vertical axis as the charges.

   [3 marks]

   (iii) Write a formula for C, in terms N, that can be used by the local council to calculate their charges.

   [1 mark]

(b) The local hotel calculates charges for their conference room using the formula:

$$C = \frac{5N}{2} + 500$$

where C is the charge in £ and N is the number of guests.

   (i) Describe, **in words only**, what this formula means.

   [2 marks]

   (ii) **Copy** and complete this table for the charges made by the hotel.

Number of guests (N)	0	20	40	80
Charges (C) in £				

   [2 marks]

   (iii) On the same axes used in part (a)(ii), draw this graph of C. Label your graph clearly.

   [2 marks]

(This question continues on the following page)

(Question 5 continued)

    (c) Explain, briefly, what the two graphs tell you about the charges made.   *[2 marks]*

    (d) Using your graphs or otherwise, find

        (i) the cost of renting the community hall if there are 87 guests;   *[2 marks]*

        (ii) the number of guests if the hotel charges £650 ;   *[2 marks]*

        (iii) the difference in charges between the council and the hotel if there are 82 guests at the reception.   *[2 marks]*

6.   *[Maximum mark: 20]*

Two types of minibus, Kombi and Danfo, are used to transport 70 students from their hall of residence to college each day. The college can provide up to 7 drivers but no driver is allowed to repeat the journey. The following additional information is known:

Type of Minibus	Number of minibuses available	Maximum number of students
Kombi	4	15
Danfo	5	10

(a)   Let x and y represent the number of Kombi and Danfo minibuses, respectively, used each day.

    (i) Explain why $x + y \leqslant 7$.   *[2 marks]*

    (ii) Write down and simplify an inequality in x and y which ensures that all 70 students are transported.   *[2 marks]*

    (iii) Explain why the two inequalities $x \leqslant 4$ and $y \leqslant 5$ are valid.   *[2 marks]*

(b)   (i) Draw a graph to show the region defined by the four inequalities in part (a). Use a suitable scale for each axis.   *[5 marks]*

    (ii) Indicate, **by shading**, the region defined by the four inequalities.   *[2 marks]*

    (iii) From your graph write down all **possible** combinations of Kombi and Danfo minibuses that simultaneously satisfy all four inequalities.   *[2 marks]*

The cost of using a Kombi minibus is £23 a day and the cost of using a Danfo minibus is £15 a day.

(c)   (i) Write an expression in terms of x and y for the total cost, C, of transporting the students each day.   *[1 mark]*

    (ii) **Using your answer to part (b)(iii)**, determine the minimum daily cost of transporting the 70 students. Show your method clearly.   *[3 marks]*

    (iii) On a particular day, one of the Kombi minibuses is in the garage for repair. What is the cheapest way of transporting the students on that day?   *[1 mark]*

7.   *[Maximum mark: 20]*

(i) A ship sails from port A and travels due South to port B.

From port B it sails on a bearing of 060° and travels for 50 km to a point C , which is due East of A .

(a)   (i) Draw, and label clearly, a diagram to show A , B and C .   *[2 marks]*

       (ii) Calculate the distance from port A to point C .   *[2 marks]*

A second ship also sails from port A , but on a bearing of 330° for 50 km to a point D .

(b)   Complete your diagram in part (a) to show point D .   *[2 marks]*

(c)   Calculate

      (i) the distance from point C to point D ;   *[3 marks]*
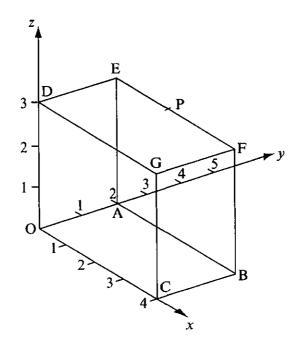
      (ii) angle ACD .   *[3 marks]*

(d)   What is the bearing of D from C ?   *[2 marks]*

(This question continues on the following page)

(*Question 7 continued*)

(ii) A rectangular prism OABCDEFG is drawn on 3-dimensional axes as shown below.

Point P is the midpoint of the edge EF .

(a) Write down the coordinates of point P .   *[1 mark]*

(b) Calculate, **correct to three significant figures**

    (i) the length of the internal diagonal EC ;   *[3 marks]*

    (ii) angle AEC .   *[2 marks]*